PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY2201**
Name of module	**Statistical Physics**
Date of examination	**Jan 2007**

1. i) a) infinity; b) zero.

   ii) See course notes.

   iii) Expansion is irreversible, so $\mathrm{d}S > \delta Q / T$.

   iv) Bookwork.

   v) $\Delta S = \int_{T_1}^{(T_1+T_2)/2} \frac{C_V\,\mathrm{d}T}{T} + \int_{T_2}^{(T_1+T_2)/2} \frac{C_V\,\mathrm{d}T}{T} = C_V \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right] = C_V \ln\left[\frac{4T_1T_2+(T_1-T_2)^2}{4T_1T_2}\right] > 0.$

2. See course notes.
   Isotropy implies $p(-u_x) = p(u_x)$.
   See course notes. Note, that the number of states with the speed from u to $u+\mathrm{d}u$ is $2\pi u\,\mathrm{d}u$.
   $\varepsilon = mu^2/2 \Rightarrow u\,\mathrm{d}u = \frac{1}{m}\mathrm{d}\varepsilon \quad \Rightarrow \; p(u)\mathrm{d}u = 2\frac{\alpha}{m}\exp(-2\alpha\varepsilon/m)\mathrm{d}\varepsilon = \beta\exp(-\beta\,\varepsilon)\mathrm{d}\varepsilon.$
   Density of states in 2D does not depend on energy.

3. i) See course notes. In case of non-degenerate levels, $p_i = \exp\left(-{\varepsilon_i}/{k_\mathrm{B}T}\right) \Big/ \sum_j \exp\left(-{\varepsilon_j}/{k_\mathrm{B}T}\right)$.

   ii) $p = 5 \Big/ \left\{\frac{(6+5-1)!}{6!\,(5-1)!}\right\} = \frac{5}{210} \approx 0.024$; $S = k_\mathrm{B}\ln 5 \approx 1.6\,k_\mathrm{B} \approx 2.2\times10^{-23}\,\mathrm{J\,K^{-1}}$.

   iii) See course notes.

4. i) See course notes.

   ii) See course notes.

   iii) $\left(\frac{\partial P}{\partial V}\right)_T \left(\frac{\partial V}{\partial T}\right)_P \left(\frac{\partial T}{\partial P}\right)_V = -1 \Rightarrow \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P = -\frac{1}{V}\left(\frac{\partial V}{\partial P}\right)_T \left(\frac{\partial P}{\partial T}\right)_V$ or $\alpha_P = -\beta_V \kappa_T$.

5. i) $S = k_\mathrm{B}\ln\Omega,\; \Omega_{A+B} = \Omega_A \cdot \Omega_B \Rightarrow S_{A+B} = k_\mathrm{B}\ln\Omega_A + k_\mathrm{B}\ln\Omega_B = S_A + S_B$.

   ii) Maximum work implies $\Delta S_{total} = \Delta S_A + \Delta S_B + \Delta S_C = 0.$
   Therefore, $C\left\{\int_{T_A}^{T_f}\frac{\mathrm{d}T}{T} + \int_{T_B}^{T_f}\frac{\mathrm{d}T}{T} + \int_{T_C}^{T_f}\frac{\mathrm{d}T}{T}\right\} = 0$, so $\ln\left[T_f^3/(T_AT_BT_C)\right] = 0 \Rightarrow T_f = (T_AT_BT_C)^{1/3}$.

   iii) $Z = 1\cdot\exp(0) + 3\cdot\exp(-1) + 5\cdot\exp(-2) \approx 2.78$.