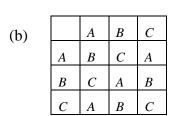
## PHYSICS EXAMINATION PROBLEMS SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	PHY3140		
Name of module	Methods of theoretical physics		
Date of examination	May/June 2010		

E.g., classification of vibrational modes or selection rules for optical transitions. 1. (i)

	(1	0	0)
(a) $\underline{\underline{C}} = \underline{\underline{A}}\underline{\underline{B}} =$	0	1	0 .
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(c)  $\underline{\underline{E}} = \underline{\underline{C}}$ . (d) Multiplication table is symmetric ( $\underline{\underline{AB}} = \underline{\underline{BA}}$ ), therefore the group is Abelian. All elements can be expressed as  $\underline{\underline{A}}^k$ , where k = 1,2,3,  $\Rightarrow$  the group is cyclic. (e)  $\underline{\underline{A}}^{-1} = \underline{\underline{B}}$ .

- (ii) (a)  $\int_{0}^{2\pi} \frac{d\theta}{a + \sin \theta} = \frac{2\pi}{\sqrt{a^2 1}}$ . Hint: Substitute  $z = \exp(i\theta)$  and use contour integration.
  - (b)  $\int_{0}^{+\infty} \frac{dx}{(x^2+1)^3} = \frac{3\pi}{16}$ . Hint: Contour integration and the residue formula for a third-order pole.
- (i)  $\lambda = -1$ . w(x, y) = 2xy + A, where A is a real number.  $f(z) = z^2 + iA$ .

(ii) 
$$E = \left(\frac{\pi}{2\sqrt{2}\gamma}\right)^{5/3} \left(\frac{\alpha\hbar^{10}}{m^5}\right)^{1/6} \left(n + \frac{1}{2}\right)^{5/3} \approx 1.32 \times \left(\frac{\alpha\hbar^{10}}{m^5}\right)^{1/6} \left(n + \frac{1}{2}\right)^{5/3}.$$
Here  $\gamma = \int_0^1 \sqrt{1 - t^{10}} \, dt \approx 0.94$ .

- 3. (i) Taylor series:  $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)n!}$ . [Integrate the integrand's Taylor expansion.] Asymptotic expansion: erf  $(x) = 1 - \frac{1}{\sqrt{\pi}} \left[ \frac{1}{x} - \frac{1}{2x^3} + \frac{1 \cdot 3}{2^2} \frac{1}{x^5} - O\left(\frac{1}{x^7}\right) \right]$ . [Use by-parts integration] The error function is used for data analysis.
  - (ii) P = (N M)/N. Bookwork (definition).  $x_c(4) = 5/12$ .

4. 
$$A = 2b^{3/2}$$
.  $E = \left(\frac{3}{2}\right)^{5/3} \left(\frac{\hbar^2 F^2}{m}\right)^{1/3}$ .