PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY3140**
Name of module	**Methods of theoretical physics**
Date of examination	**June 2009**

1. (i) Fourier transform: $F(k) = \exp(-kx_0 - |k|\gamma)/\sqrt{2\pi}$.

(ii) Notations: $A = 1$; $B = (1+i)/\sqrt{2} = \exp(i\pi/4)$; $C = i$; $D = (-1+i)/\sqrt{2} = \exp(3i\pi/4)$; $F = -1$; $G = (-1-i)/\sqrt{2} = \exp(5i\pi/4)$; $H = -i$; $J = (1-i)/\sqrt{2} = \exp(7i\pi/4)$.

(a)

	A	**B**	**C**	**D**	**F**	**G**	**H**	**J**
A	*A*	*B*	*C*	*D*	*F*	*G*	*H*	*J*
B	*B*	*C*	*D*	*F*	*G*	*H*	*J*	*A*
C	*C*	*D*	*F*	*G*	*H*	*J*	*A*	*B*
D	*D*	*F*	*G*	*H*	*J*	*A*	*B*	*C*
F	*F*	*G*	*H*	*J*	*A*	*B*	*C*	*D*
G	*G*	*H*	*J*	*A*	*B*	*C*	*D*	*F*
H	*H*	*J*	*A*	*B*	*C*	*D*	*F*	*G*
J	*J*	*A*	*B*	*C*	*D*	*F*	*G*	*H*

(b) The unit element is $A = 1$. (c) Multiplication table is symmetric ($AB = BA$, etc), thus, the group is Abelian. All elements can be expressed as B^k, where $k = 0,1,2,3,4,5,6,7$, therefore, the group is cyclic. (d) The inverse element of J is B.

2. (i) $2\pi/\sqrt{1-a^2}$. Hint: Substitute $z = \exp(i\theta)$ and use contour integration.

(ii) (a) $P_2(x) = x\left[1-(1-x)^3\right]$. (b) $P_2(x) = x\left[1-(1-x)^6\right]$. (c) $P_2(x) = x\left[1-(1-x)^d\right]$.

3. (a) $E = \dfrac{25}{8\times(858)^{1/5}}\left(\dfrac{\hbar^8\alpha}{m^4}\right)^{1/5} \approx 0.93\left(\dfrac{\hbar^8\alpha}{m^4}\right)^{1/5}$. (b) $E = \dfrac{5\times(1260)^{1/5}}{8}\left(\dfrac{\hbar^8\alpha}{m^4}\right)^{1/5} \approx 2.61\left(\dfrac{\hbar^8\alpha}{m^4}\right)^{1/5}$.

Function (a) is a better approximation.

4. (i) $\beta = \pm\alpha$. $w = \mp\exp(\alpha x)\cos(\alpha y) + \text{const}$.

(ii) $$E = \left(\frac{\pi}{2\sqrt{2}\gamma}\right)^{8/5}\left(\frac{\hbar^8\alpha}{m^4}\right)^{1/5}\left(n+\frac{1}{2}\right)^{8/5} \approx 1.33\left(\frac{\hbar^8\alpha}{m^4}\right)^{1/5}\left(n+\frac{1}{2}\right)^{8/5}.$$

Here $\gamma = \int_0^1 \sqrt{1-t^8}\,\mathrm{d}t \approx 0.93$.