PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY3140**
Name of module	**Methods of theoretical physics**
Date of examination	**June 2007**

1. (i) (a) Poles are at $z = \exp(i\pi/4)$, $z = \exp(i3\pi/4)$, $z = \exp(i5\pi/4)$ and $z = \exp(i7\pi/4)$.

(b) Upper half of z - plane: residue of $f(z)$ at the pole $z = \exp(i\pi/4)$ is $\dfrac{\exp(-i3\pi/4)}{4}$;

residue at $z = \exp(i3\pi/4)$ is $\dfrac{\exp(-i9\pi/4)}{4}$.

(c) $\displaystyle\int_0^{+\infty} \frac{1}{x^4+1} \mathrm{d}x = \frac{\pi\sqrt{2}}{4}$.

(ii) (a)

	1	-1	*i*	-*i*
1	1	-1	*i*	-*i*
-1	-1	1	-*i*	*i*
I	*i*	-*i*	-1	1
-*i*	-*i*	*i*	1	-1

(b) $E = 1$.

(c) Multiplication table is symmetric ($AB = BA$), therefore the group is Abelian.
All elements can be expressed as i^k , where $k = 0,1,2,3$, therefore the group is cyclic.

(d) The inverse element of $-i$ is i.

2. (i) $\displaystyle\int_0^{2\pi} \frac{d\theta}{a + b\sin\theta} = \frac{2\pi}{\sqrt{a^2 - b^2}}$ (for $0 < b < a$). [Hint: Substitute $z = \exp(i\theta)$ and use contour integration.]

(ii) (a) $P_2(x) = x\left[1-(1-x)^4\right] \approx 4x^2$ (for $x << 1$); (b) $P_2(x) = x\left[1-(1-x)^6\right] \approx 6x^2$ (for $x << 1$);
(c) $P_2(x) = x\left[1-(1-x)^z\right] \approx zx^2$ (for $x << 1$).

3. (a) $A = 2b^{3/2}$. (b) $E = \left(\dfrac{3}{2}\right)^{5/3} \left(\dfrac{\hbar^2 F^2}{m}\right)^{1/3}$.

4. (i) (a) $\beta = \pm\alpha$. (b) $u = \dfrac{\alpha}{\beta}\exp(\alpha x)\cos(\beta y) + const$, with $\beta^2 = \alpha^2$.

(ii) $E_n = \left(\dfrac{9\pi^2}{32}\right)^{1/3} \left(n + \dfrac{1}{2}\right)^{2/3} \left(\dfrac{\hbar^2 F^2}{m}\right)^{1/3}$.