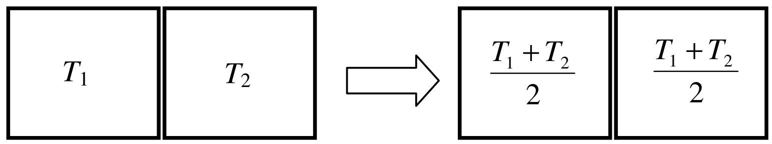
PHY2023 Supplement 1: Entropy change of two identical bodies reaching thermal equilibrium.

Consider two identical bodies, heat capacity c_v, initially at different temperatures T_1 and T_2. We will show that the process of reaching thermal equilibrium necessarily involves a total increase in entropy.



If heat loss to the external environment is made negligible and if the heat capacity of the blocks is independent of temperature, then if the block at T_1 cools by ΔT_1 through transferring heat ΔQ_1 whilst the block at T_2 heats by ΔT_2 through transferring heat ΔQ_2 then

$$\Delta Q_1 = -\Delta Q_2 \qquad \text{(1}^{\text{st}}\text{ law of thermodynamics)}$$
$$T_1 - \Delta T_1 = T_2 - \Delta T_2 \qquad \text{(2}^{\text{nd}}\text{ law of thermodynamics)}$$

$$\therefore c_v \Delta T_1 = -c_v \Delta T_2$$
$$\therefore T_1 - \Delta T_1 = T_2 + \Delta T_1$$
$$\therefore \Delta T_1 = \frac{T_1 - T_2}{2}$$
$$\therefore T_1 - \Delta T_1 = T_2 - \Delta T_2 = \frac{T_1 + T_2}{2}$$

Since entropy is a function of state, we can calculate the entropy change of each block by considering an ideal, reversible process taking each block from its initial temperature to a final temperature of $(T_1+T_2)/2$. The entropy of the surroundings can be ignored as we are assuming that heat loss to the surroundings is negligible. Hence

$$\Delta S = \Delta S_1 + \Delta S_2 = \int_{T_1}^{\frac{T_1+T_2}{2}} \frac{c_v \mathrm{d}T}{T} + \int_{T_2}^{\frac{T_1+T_2}{2}} \frac{c_v \mathrm{d}T}{T}$$
$$= c_v \left[\ln\left(\frac{T_1+T_2}{2}\right) - \ln(T_1) + \ln\left(\frac{T_1+T_2}{2}\right) - \ln(T_2) \right]$$
$$= c_v \ln\left(\frac{(T_1+T_2)^2}{4T_1T_2}\right)$$
$$= c_v \ln\left(1 + \frac{(T_1-T_2)^2}{4T_1T_2}\right)$$

Note that if $T_1 \neq T_2$ then $\Delta S > 0$.