PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY2201**
Name of module	**Statistical Physics**
Date of examination	**Jan 2011**

1. (i) $\Delta S_{universe}^{reversible} = 0$. Melting ice: $\Delta S = mL_f/T = 1220$ JK^{-1}. Heating water: $\Delta S = k_B \ln(T_2/T_1) = 1311$JK^{-1}.

(ii) $\delta Q = 0 \Rightarrow C_V \mathrm{d}T + \frac{A}{V^2}\mathrm{d}V + P\mathrm{d}V \Rightarrow C_V \frac{\mathrm{d}T}{T} = -R\frac{\mathrm{d}V}{V-B} \Rightarrow \ln[T^{C_V}(V-B)^R] = const \Rightarrow (RT)^{C_V}(V-B)^R = const$

$\Rightarrow \left(P + \frac{A}{V^2}\right)^{C_V}(V-B)^{C_V+R} = const \Rightarrow \left(P + \frac{A}{V^2}\right)(V-B)^{\gamma} = const$, where $\gamma = \frac{C_V+R}{C_V}$.

2. See course notes. Isotropy implies $p(-u_x) = p(u_x)$.

See course notes. Note, that the number of states with the speed from u to $u + \mathrm{d}u$ is $2\pi u\,\mathrm{d}u$.

$$\varepsilon = mu^2/2 \Rightarrow u\,\mathrm{d}u = \frac{1}{m}\mathrm{d}\varepsilon \quad \Rightarrow \quad p(u)\mathrm{d}u = 2\frac{\alpha}{m}\exp(-2\alpha\varepsilon/m)\mathrm{d}\varepsilon = \beta\exp(-\beta\varepsilon)\mathrm{d}\varepsilon.$$

Boltzmann factor: $\exp(-\beta\varepsilon)$. $\langle\varepsilon\rangle = \int_0^\infty \varepsilon\, p(\varepsilon)\,\mathrm{d}\varepsilon = \frac{1}{\beta} = 2\frac{k_B T}{2} \Rightarrow \beta = \frac{1}{k_B T}$.

Density of states in 2D does not depend on energy. $p(\varepsilon < k_B T) = \int_0^{k_B T} p(\varepsilon)\,\mathrm{d}\varepsilon = \int_0^1 e^{-t}\,\mathrm{d}t = (e-1)/e.$

3. See course notes. $p_i = \exp\left(-\frac{\varepsilon_i}{k_B T}\right) \Big/ \sum_l \exp\left(-\frac{\varepsilon_l}{k_B T}\right)$. $Z = \sum_l \exp\left(-\frac{\varepsilon_l}{k_B T}\right)$ ensures $\sum_i p_i = 0.$

$$\overline{U} = \sum_r \varepsilon_r p_r = \frac{1}{Z}\sum_r \varepsilon_r \exp\left(-\frac{\varepsilon_r}{k_B T}\right) = -\frac{1}{Z}\frac{\mathrm{d}}{\mathrm{d}\beta}\sum_r \exp(-\beta\varepsilon_r) = -\frac{1}{Z}\frac{\mathrm{d}Z}{\mathrm{d}\beta} = -\frac{1}{Z}\frac{\mathrm{d}\ln(Z)}{\mathrm{d}\beta}.$$

$$Z = g_0\exp\left(-\frac{\varepsilon_0}{k_B T}\right) + g_1\exp\left(-\frac{\varepsilon_1}{k_B T}\right) + g_2\exp\left(-\frac{\varepsilon_2}{k_B T}\right) = 1 + 3e^{-1} + 5e^{-2} \approx 2.78.$$

$$\overline{U} = \frac{1}{Z}\sum_r \varepsilon_r g_r \exp\left(-\frac{\varepsilon_r}{k_B T}\right) = k_B T\left(0 + 1\times 3e^{-1} + 2\times 5e^{-2}\right)/Z \approx 0.88\,k_B T.$$

4. (i) $H = U + PV$. $\mathrm{d}H = T\mathrm{d}S + V\mathrm{d}P \Rightarrow \mathrm{d}H = T\mathrm{d}S = \delta Q$ (heat flow) for $P = const$.

(ii) See course notes: $F = U - TS$; extensive; $\mathrm{d}F = -P\,\mathrm{d}V - S\,\mathrm{d}T \Rightarrow \mathrm{d}F = -\delta W$ for $T = const$.

(iii) $\Delta S_{total} = 0 \Rightarrow C\left(\ln\frac{T_f}{T_A} + \ln\frac{T_f}{T_B} + \ln\frac{T_f}{T_C} + \ln\frac{T_f}{T_D} + \ln\frac{T_f}{T_E}\right) = C\ln\left[T_f^5/(T_A T_B T_C T_D T_E)\right] = 0$

$\Rightarrow T_f^5/(T_A T_B T_C T_D T_E) = 1 \Rightarrow T_f = (T_A T_B T_C T_D T_E)^{1/5}$.

5. (i) $S = k_B \ln\Omega$; see course notes. $\Omega_{A+B} = \Omega_A\cdot\Omega_B \Rightarrow S_{A+B} = k_B\ln\Omega_A + k_B\ln\Omega_B = S_A + S_B$

(ii) a) $\Omega_0 = 1 \Rightarrow S = 0$; b) $\Omega_1 = N = 10 \Rightarrow S = k_B \ln N = k_B \ln 10 \approx 3.18\times 10^{-23}$ JK^{-1};

c) $S = k_B\ln\Omega_4 = k_B\ln\left(\frac{N!}{4!\,(N-4)!}\right) = k_B\ln\left(\frac{10!}{4!6!}\right) = k_B\ln(210) \approx 7.38\times 10^{-23}$ JK^{-1}.

d) Most probable: $N_{heads} = N_{tales} = 5 \Rightarrow S = k_B\ln\Omega_5 = k_B\ln\left(\frac{10!}{5!5!}\right) = k_B\ln(252) \approx 7.63\times 10^{-23}$ JK^{-1}.

(iii) See course notes.

Classical limit: $\exp\left(\frac{E_i - E_F}{k_B T}\right) >> 1$. Therefore, $f(E_i) \propto \exp\left(-\frac{E_i}{k_B T}\right)$ – Boltzmann's factor.