PHYSICS EXAMINATION PROBLEMS
SOLUTIONS AND HINTS FOR STUDENT SELF-STUDY

Module Code	**PHY2201**
Name of module	**Statistical Physics**
Date of examination	**Jan 2008**

1. i) See course notes. $\Delta S_{total} = -\frac{Q_1}{T_1} + \frac{Q_2}{T_2} \Rightarrow$ if $Q_2 = 0 \Rightarrow \Delta S < 0,$ contrary to the 2nd Law, unless $T = 0$.

   ii) $P_{\min} = Q_2 \frac{T_1 - T_2}{T_2} \approx 3\,\text{MW}$.

   iii) Use $\mathrm{d}S_1 = \frac{m c_V \mathrm{d}T}{T}$ for water and $\Delta S_2 = -m c_V \frac{\Delta T}{T_f}$ for the reservoir, where $T_f = T_{reservoir}$

   $$\Delta S_{total} = m c_V \left[\ln\left(\frac{T_f}{T_i}\right) + \frac{T_i}{T_f} - 1 \right] \approx 122\ \mathrm{J\,K^{-1}}.$$

2. i) See course notes. Equilibrium: $\langle v_x \rangle = \langle v_y \rangle = \langle v_z \rangle = 0 \Rightarrow$ no bulk motion..

   ii) See course notes.

   $$\langle \varepsilon \rangle = \int_0^\infty \varepsilon\, p(\varepsilon)\, \mathrm{d}\varepsilon = \frac{3}{2\beta} = \frac{3k_\mathrm{B}T}{2} \Rightarrow \beta = \frac{1}{k_\mathrm{B}T}.$$

   Conduction electrons – ‘quantum gas’, rules governing occupancy dominate behaviour.

3. i) See course notes.

   ii) $T = \frac{\Delta\varepsilon}{k_\mathrm{B} \ln 10} = 2.5 \times 10^4$ K.

   iii) $\Omega_{n,k} = \frac{(n+k-1)!}{n!k!} = 35.$ Equally shared energy: $p = \frac{1}{35} \approx 0.029;$ $S = 0.$ Energy shared between two systems: $p = \frac{6}{35} \approx 0.17.$

4. i) See course notes.

   ii) See course notes.

   iii) Use $Z = \sum_{i=0}^{\infty} \exp\left(-\frac{\varepsilon_i}{k_\mathrm{B}T}\right)$ and the geometrical series summation rule.

5. i) $S = k_\mathrm{B} \ln \Omega$; see course notes. $\Omega_{A+B} = \Omega_A \cdot \Omega_B \Rightarrow S_{A+B} = k_\mathrm{B} \ln \Omega_A + k_\mathrm{B} \ln \Omega_B = S_A + S_B$

   a) $S = 0$: b) $S = k_\mathrm{B} \ln N \approx 3.2 \times 10^{-23}\ \mathrm{JK^{-1}}$;

   c) $S = k_\mathrm{B} \ln \Omega_{N/2} = k_\mathrm{B} \ln \frac{N!}{(N/2)!(N/2)!} \approx N k_\mathrm{B} \ln 2 \approx 7.63 \times 10^{-23}\ \mathrm{JK^{-1}}$.

   ii) See course notes.

   Classical limit: $\exp\left(\frac{E_i - E_\mathrm{F}}{k_\mathrm{B}T}\right) >> 1$. Therefore, $\frac{n_i}{w_i} \propto \exp\left(-\frac{E_i}{k_\mathrm{B}T}\right)$.