Fourier Analysis Problem Set 1

1. Prove that an odd function can contain no cosine terms in its full range Fourier expansion. (Break the integral for the coefficient a_n into two parts for $x < 0$ and $x > 0$, and show that these are equal and opposite). [4]

2. Find the Fourier series expansion of

$$f(x) = \begin{cases} 1, & |x| < \dfrac{L}{2} \\ 0, & \dfrac{L}{2} < |x| < L \end{cases}$$

in the interval $(-L, L)$. Hence show that

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots$$

[9]

3. Show that the Fourier series expansion of

$$f(x) = \begin{cases} x, & 0 < x < L \\ -x, & -L < x < 0 \end{cases}$$

in the interval $(-L, L)$ may be written as

$$f(x) = \frac{L}{2} - \frac{4L}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} \cos\left(\frac{\pi(2n-1)x}{L}\right).$$

[9]

4. (a) By making a Fourier series expansion of

$$f(x) = \begin{cases} \sin x, & 0 \le x \le \pi \\ 0, & -\pi \le x \le 0 \end{cases}$$

in the interval $(-\pi, \pi)$, show that for $0 \le x \le \pi$

$$\sin x = \frac{4}{\pi}\left[\frac{1}{2} - \frac{\cos 2x}{1\cdot 3} - \frac{\cos 4x}{3\cdot 5} - \frac{\cos 6x}{5\cdot 7} - \ldots\right].$$

(b) Use the series to show that

$$\frac{\pi}{4} = \frac{1}{2} + \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \ldots.$$

[14]

5. The Fourier expansion of a function $f(x)$ in the interval $(-L, L)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right).$$ [6]

By squaring both sides of the equation and integrating from $-L$ to L, prove that

$$\frac{1}{L}\int_{-L}^{L}(f(x))^2\,dx = \frac{a_0^2}{2} + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)$$

(This result is known as Parseval's identity)

6. A low pass filter passes all frequencies below 800 Hz and removes all of those above 800 Hz. Make a sketch of how a square wave signal of period 10 ms would look after passing through the filter and describe its form in detail. (Hint: begin by find the Fourier expansion of the square wave)

[7]