Differential Calculus Problem Set 3

1. Verify Green's Theorem in the plane for $\oint_C \left[\left(x^2 - xy^3\right)dx + \left(y^2 - 2xy\right)dy\right]$, where C is a rectangle with vertices at $(-1,-2)$, $(1,-2)$, $(1,1)$ and $(-1,1)$. [15]

2. Determine the surface area of the plane $2x + y + 4z = 16$ cut off by (a) $x = 0$, $y = 0$, $x = 2$, $y = 3$, (b) $x^2 + y^2 = 64$. Begin by sketching the surface in each case. [6]

3. A sphere of radius a has its centre at the origin. The charge per unit area on the sphere is given by $\sigma = x^2 + 2y^2$. Find the total charge on the sphere. [7]

4. Evaluate $\int_{-5}^{5}\int_{-5}^{5} 5xy\delta(x-2)\delta(y+3)\,dxdy$. [4]

5. By considering $\int_{-\infty}^{\infty} f(x)\frac{d(\delta(x))}{dx}dx$ and $\int_{-\infty}^{\infty} f(x)\frac{\delta(x)}{x}dx$ verify that $\frac{d}{dx}\delta(x) = -\frac{1}{x}\delta(x)$. (Hint: make a Taylor expansion of function f as a function of x about $x = 0$ and note that $\int_{-\infty}^{+\infty}\frac{\delta(x)}{x}dx = 0$ since the integrand is an odd function of x). [9]

6. Show that the value of the integral $\int_{-\infty}^{+\infty}\frac{n}{\pi\left(1+n^2x^2\right)}dx$ does not depend upon the value of n. [4]