

# Expansion of Liquid $^4\text{He}$ Through the Lambda Transition

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*Zurek suggested (Nature **317**, 505; 1985) that the Kibble mechanism, through which topological defects such as cosmic strings are believed to have been created in the early Universe, can also result in the formation of topological defects in liquid  $^4\text{He}$ , i.e. quantised vortices, during rapid quenches through the superfluid transition. Preliminary experiments (Hendry et al, Nature **368**, 315; 1994) seemed to support this idea in that the quenches produced the predicted high vortex-densities. The present paper describes a new experiment incorporating a redesigned expansion cell that minimises vortex creation arising from conventional hydrodynamic flow. The post-quench line-densities of vorticity produced by the new cell are no more than  $10^{10}\text{m}^{-2}$ , a value that is at least two orders-of-magnitude less than the theoretical prediction. We conclude that most of the vortices detected in the original experiment must have been created through conventional flow processes.*

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## 1. INTRODUCTION

After a physical system has passed rapidly through a continuous phase-transition, its order-parameter can have components with large differences between adjacent, but causally disconnected, “domains”. In such systems topological defects can form at the domain boundaries.<sup>1</sup> This idea was proposed by Kibble<sup>2</sup> in connection with the grand unified theory (GUT) symmetry-breaking phase-transition of the early Universe, and has been developed by Zurek<sup>3,4,5</sup> who has estimated how the density of defects created

depends on the rate at which the system passes through the transition. Zurek also pointed out that this mechanism of defect production was applicable, in principle, to all continuous phase-transitions, and that it should therefore be possible to validate some aspects of cosmological theories through laboratory-scale experiments. The first of these examined weakly first-order phase-transitions in liquid crystals.<sup>6,7</sup> Later, the corresponding experiments were carried out using the second-order superfluid phase-transitions of liquid  $^4\text{He}$ <sup>8</sup> and liquid  $^3\text{He}$ .<sup>9,10</sup> All these experiments produced defect densities reported as being consistent with Zurek's estimates.<sup>3,4,5</sup>

In this paper we describe an improved version of the  $^4\text{He}$  experiment<sup>8</sup> in which particular care has been taken to minimise the production of topological defects (*i.e.* quantised vortices) by ordinary hydrodynamic flow processes. Our new results, of which a preliminary report<sup>11</sup> has already been published, show no convincing evidence of *any* vortex creation at all. But they allow us to place an approximate upper-bound on the initial density of vortices produced by the Kibble-Zurek mechanism.

## 2. THEORETICAL BACKGROUND

A considerable amount is known about the properties<sup>12,13,14,15</sup> of vortices, and about how they are created at very low temperatures<sup>16,17,18</sup> but the mechanism responsible for the vorticity that appears<sup>19,8</sup> as a result of passing through the  $\lambda$ -transition (which separates the normal helium-I and the superfluid helium-II phases) is not understood. One possibility is that pre-existing rotational flow caused by e.g. convection or boiling in the helium-I phase is, with the onset of long-range order, converted into quantised vortex lines in helium-II; this is quite distinct from the Zurek scenario which we shall now briefly describe.

The underlying idea<sup>3</sup> is quite simple. A small isolated volume of helium-I is initially held at pressure  $P_i$ , and temperature  $T_i$ , just above the temperature  $T_\lambda(P_i)$  of the  $\lambda$ -transition. The logarithmic infinity in its heat capacity at  $T_\lambda$  makes it impossible to *cool* the sample quickly into the superfluid phase but the pressure dependence of  $T_\lambda$  means that it can be taken through the transition very rapidly by adiabatic *expansion* into the helium-II phase, to a final pressure  $P_f$  and temperature  $T_f$  (Fig 1). Fluctuations present in the helium-I are expected to cause the nascent superfluid to form with a spatially incoherent order-parameter, corresponding to a large density of vortex lines. This scenario depends on the fact that the liquid can, in principle, expand at velocities comparable to that of first-sound, whereas the propagation velocity for changes in the order-parameter is limited by the much slower velocity of second-sound.

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The analogy<sup>3,4,5</sup> between liquid helium and the early Universe arises because they both be considered to undergo second-order phase transitions describable in terms of Ginsburg-Landau theory.<sup>20</sup> In each case the potential contribution to the free-energy density can be written as:

$$V = \alpha(T)|\psi|^2 + \frac{1}{2}\beta|\psi|^4 \quad (1)$$

where the parameter  $\alpha$  is positive at temperatures above  $T_\lambda$  and negative below it, and  $\beta$  is a constant. For liquid  $^4\text{He}$  the order-parameter is the modulus of the complex-scalar field  $\psi$ , *i.e.* the Bose condensate wave-function which is a solution of the Ginsburg-Pitaevskii equation.<sup>20</sup> In the cosmological analogy the components of  $\psi$  are Higgs fields.<sup>4,5</sup> In the symmetric (helium-I) phase  $T > T_\lambda$  and the time-average of the order-parameter  $\langle\psi\rangle = 0$ . Below  $T_\lambda$  this gauge symmetry, is broken so  $\langle\psi\rangle$  becomes non-zero, and the real and imaginary parts of the potential in equation 1 acquire the same “sombbrero” shape as the corresponding cosmological free energy expressed in terms of Higgs fields (Fig. 2). In the early Universe a symmetry-breaking phase-transition from a false-vacuum state to a true-vacuum state is thought to have occurred once the temperature had fallen to  $\sim 10^{27}$  K, about  $10^{-35}$  s after the big bang. Although there are many variants of the basic model, with and without inflation, it is believed that a variety of topological defects<sup>2</sup> would have been produced in the transition because an event horizon prevented adjacent regions from being causally connected. Cosmic strings<sup>21</sup> — thin tubes of false vacuum — are one such defect and may have had a role in galaxy formation. It is these that correspond to the quantised vortices found in helium-II. The analogy between the helium and cosmological phase transitions may thus be summarised as follows –

Higgs field 1	$\longleftrightarrow$	Re $\psi$
Higgs field 2	$\longleftrightarrow$	Im $\psi$
False vacuum	$\longleftrightarrow$	He I
True vacuum	$\longleftrightarrow$	He II
Cosmic string	$\longleftrightarrow$	quantized vortex line

### 3. DETECTING VORTICITY

In our initial experiments,<sup>8,23</sup> the vortex density was measured by recording the attenuation of a sequence of second-sound pulses propagated through the helium-II. We expected that, following an expansion, the pulse amplitude would grow towards its vortex-free value as the tangle decayed and the attenuation decreased.

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A considerable amount is known about the decay<sup>13,14,15,24</sup> of hydrodynamically created vortex tangles in helium-II. Numerical simulations<sup>24,25</sup> give a good qualitative description of the manner in which a homogeneous isotropic tangle evolves and decays. The rate at which it occurs in this temperature range is governed by the Vinen<sup>14</sup> equation

$$\frac{dL}{dt} = -\chi_2 \frac{\hbar}{m_4} L^2 \quad (2)$$

where  $L$  is the vortex-line density at time  $t$ , the  $^4\text{He}$  atomic mass is  $m_4$ , and  $\chi_2$  is a dimensionless parameter. The relationship between vortex density and second-sound attenuation is known<sup>15</sup> from experiments with rotating helium, and may for present purposes be written

$$L = \frac{6c_2}{\kappa Bd} \ln(S_0/S) \quad (3)$$

where  $c_2$  is the velocity of second-sound,  $S$  and  $S_0$  are the signal amplitudes with and without vortices present respectively.  $B$  is a weakly temperature dependent parameter,  $\kappa = h/m_4$  is the quantum of circulation, and  $d$  is the transducer separation. Integrating equation 2 and substituting for  $L$  from equation 3 gives an expression for the recovery of the signal:

$$\frac{1}{\ln(S_0) - \ln(S)} = \frac{6c_2}{\kappa Bd} \left( \frac{\chi_2 \kappa t}{2\pi} + \frac{1}{L_1} \right) \quad (4)$$

where  $L_1$  is the vortex density immediately after the expansion. All the constants in this expression are known, although  $\chi_2$  and  $B$  do not seem to have been measured accurately within the temperature range of interest.

#### 4. THE FIRST EXPERIMENT

A description of our first attempt to realise a bulk version<sup>4</sup> of Zurek's experiment has been given in a previous paper<sup>23</sup> but, briefly, the arrangement was as follows. A cell with phosphor-bronze concertina walls was filled, by condensing in isotopically<sup>26</sup> pure  $^4\text{He}$  through a capillary tube, and then sealed with a needle-valve (Fig. 3). The top of the cell was fixed rigidly to the cryostat but its bottom surface could be moved to compress the liquid, or released to expand it, using a pull-rod from the top of the cryostat. The cell was *in vacuo* surrounded by a reservoir of liquid  $^4\text{He}$  at  $\sim 2\text{ K}$ . A Straty-Adams capacitance gauge<sup>27</sup> recorded the pressure in the cell and carbon-resistors were used as thermometers, one in the reservoir, one on the cell. The temperature of the cell could be adjusted by means of an electrical heater and a breakable thermal link to the 2 K reservoir. A trigger mechanism on

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the mechanical linkage allowed the cell to increase its volume by  $\sim 20\%$  very rapidly under the influence of its own internal pressure.

As described above, the vortex density was inferred from the amplitude of second-sound pulses created with a thin-film heater. The signal was detected with a bolometer and passed, *via* a cryogenic FET preamplifier, to a Nicolet 1280 data processor which recorded some  $\sim 200$  pulses during 1–2 s following the expansion. The last pulses in the sequence define  $S_0$ , the signal amplitude in the (virtual) absence of vortices.<sup>28</sup> Fig. 4 shows examples of data recorded with this first version of the experiment.

There was a “dead period” of about 50 ms after the mechanical shock of the expansion which caused vibrations that obscured the signals. The initial vortex-density was therefore obtained from plots such as Fig. 4(a) by back-extrapolation to the moment  $t = 0$  of traversing the transition and was found to be  $\sim 10^{12} - 10^{13} \text{ m}^{-2}$  consistent with the theoretical expectation.<sup>3,4,5</sup> However, an unexpected observation in these initial experiments<sup>8</sup> was that small densities of vortices were created even for expansions that occurred wholly in the superfluid phase (Fig. 4c), provided that the starting point was very close to  $T_\lambda$ . The phenomenon was initially<sup>23</sup> attributed to vortices produced in thermal fluctuations within the critical regime, but it was pointed out<sup>29</sup> that effects of this kind are only to be expected for expansions starting within a few microkelvin of the transition, *i.e.* much closer than the typical experimental value of a few millikelvin. The most plausible interpretation — that the vortices in question were of conventional hydrodynamic origin, arising from non-idealities in the design of the expansion chamber — was disturbing, because expansions starting above  $T_\lambda$  traverse the same region. Thus some, at least, of the vortices seen in expansions through the transition were probably not attributable to the Zurek-Kibble mechanism as had been assumed. It has been of particular importance, therefore, to undertake a new experiment with as many as possible of the non-idealities in the original design eliminated or minimised.

## 5. THE NEW EXPERIMENT

An ideal experiment would avoid all fluid flow parallel to surfaces during the expansion. This could, in principle, be accomplished by the radial expansion of a spherical volume, or the axial expansion of a cylinder with stretchy walls. In neither of these cases would there be any relative motion of fluid and walls in the direction parallel to the walls, and hence no hydrodynamic production of vortices. However, it is impossible to eliminate the effects of fluid-flow completely from any real apparatus. At the  $\lambda$ -transition the critical velocity tends to zero, so *any* finite flow-velocity will create vor-

ticity. However, the period during which this happens is very short; the entire expansion takes only a few milliseconds so only limited growth can occur from the surface-vorticity sheet of half-vortex-rings.<sup>30</sup>

With hindsight, we can identify the principal causes of unintended vortex creation in the original experiment,<sup>8,23</sup> in order of importance, as follows: (a) expansion of liquid into the cell from the filling capillary, which was closed by a needle valve 0.5 m up-stream; (b) expansion of liquid out of a shorter capillary connecting the cell to the Straty-Adams capacitive pressure-gauge; (c) flow out of and past the fixed yoke (a U-shaped structure) on which the second-sound transducers were mounted. In addition, (d) there were the undesirable transients caused by the expansion system bouncing against the mechanical stop at room temperature. The walls of the cell were made from bronze bellows,<sup>8,23</sup> rather than being a stretchy cylinder, but the effects of the flow parallel to the convoluted surfaces were relatively small.

The design of our new expansion cell (Fig. 5) addressed all the listed problems, as follows: (a) a hydraulically-operated needle-valve eliminated the dead-volume of capillary tube; (b) the phosphor-bronze diaphragm of the pressure gauge<sup>27</sup> became an integral part of the upper cell-wall eliminating the long tube leading to the pressure gauge; (c) the cell was shortened from 25mm to  $\sim 5$ mm so that the heater-bolometer pair could be mounted flush with its top and bottom inner surfaces, thereby eliminating any need for a yoke; (d) some damping of the expansion was provided by the addition of a (light motor-vehicle) hydraulic shock-absorber.

The operation of the apparatus, the technique of data collection and the analysis were much as described previously<sup>8,23</sup> except that the rate at which the sample passed through the  $\lambda$ -transition was determined directly by simultaneous measurements of the position<sup>31</sup> of the pull-rod (giving the volume of the cell, and hence its pressure) and the temperature of the cell.

The position detector made use of the variation of inductance of a solenoid coil when a ferrite-core is moved in and out. The 1 cm long, 4 mm diameter, ferrite-core was rigidly attached to the top of the pull-rod, where it emerged from the cryostat, and was positioned so that half of it was inside the 5 mm bore of the solenoid. The solenoid itself was clamped to the cryostat top-plate so that, as the cell expanded and the pull-rod moved down, the ferrite penetrated further inside. The position was measured by making the solenoid part of a resonant LCR circuit and setting the frequency so that the output was at half maximum. The 2 MHz AC signal was rectified, smoothed and then measured using a digital oscilloscope. The response time, which was limited by the smoothing and the  $Q$  of the circuit, was  $\sim 50\mu\text{s}$  and the sensitivity was  $\sim 500$  mV/mm. This, and the linearity of the system, were measured by calibration against a micrometer. The position could in

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principle be measured to an accuracy of  $\sim \pm 2\mu\text{m}$  but in the experiment this became  $\sim \pm 10\mu\text{m}$  because of the 8 bit digitiser of the oscilloscope.

We define the dimensionless distance from the transition by

$$\epsilon = \frac{T_\lambda - T}{T_\lambda} \quad (5)$$

and the quench time  $\tau_q$  by

$$\frac{1}{\tau_q} = \left( \frac{d\epsilon}{dt} \right)_{\epsilon=0} \quad (6)$$

Fig. 6 shows a typical evolution of  $\epsilon$  with time during an expansion. In Fig. 6(a), which plots the full expansion period, it is evident that the system “bounces” momentarily near  $\epsilon = 0.02$ , but without passing back through the transition. The effect is believed to be associated with the onset of damping from the shock absorber, after backlash in the system has been taken up.

The quench time is readily determined from  $\epsilon(t)$  near the transition. In the case illustrated in Fig. 6, shown in expanded form in part (b), it was  $\tau_q = 17 \pm 1$  ms. We are thus able take both the pressure-dependence of  $T_\lambda$ , and the non-constant rate of expansion, explicitly into account.

## 6. RESULTS FROM THE NEW CELL

The improvements in the cell-design intended to eliminate vortices originating from conventional fluid-flow were clearly successful; as we had hoped, and unlike the first cell, expansion trajectories that stay within the superfluid phase, even those starting as close as 42 mK to the transition, create no detectable vorticity (Fig. 7).

We were surprised, however, to discover that no detectable vorticity was created even when the expansion trajectory passed through the transition. Signal amplitudes measured just after two such expansions are shown by the data points of Fig. 8. It is immediately evident that, unlike the results obtained from the original cell,<sup>8</sup> there is now no evidence of any systematic growth of the signal amplitude with time or, correspondingly, for the creation of any vortices at the transition. One possible reason is that the density of vortices created is smaller than the theoretical estimates,<sup>3,4,5</sup> but we must also consider the possibility that they are decaying faster than they can be measured.

## 7. THE DECAY OF A VORTEX TANGLE

To try to clarify matters, we performed a subsidiary set of experiments, deliberately creating vortices by conventional means and then following their

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decay by measurements of the recovery of the second-sound signal amplitude. By leaving the needle-valve open, so that  $\sim 0.2 \text{ cm}^3$  of liquid from the dead volume outside the needle-valve actuator-bellows squirted into the cell during an expansion, we could create large densities of vorticity and observe their decay. Despite the highly non-equilibrium situation that arises immediately following the expansion, as liquid squirts into the cell, it was found that the temperature reached a steady value after  $\sim 6 \text{ ms}$ . Fig. 9 shows two examples of attenuation plots ( $[\ln(S/S_0)]^{-1}$  against time  $t$ ) and these always, within the experimental errors, had the linear form predicted by equation 4. Fig. 10 summarises the results of a number of expansions from which it was possible to determine  $\chi_2/B$  as a function of temperature and pressure. We found that  $\chi_2/B$  was weakly temperature-dependent and over the range of interest,  $0.02 < \epsilon < 0.06$  it could be approximated by  $\chi_2/B = 0.004 \pm 0.001$ . This measured value of  $\chi_2/B$  was then used to calculate the evolution of  $S/S_0$  with time for different values of  $L_1$ , assuming  $B = 1$ , yielding the curves shown in Fig. 8. From the  $\tau_q$  derived from the gradient in Fig 6, and Zurek's estimate (based on renormalisation-group theory) of

$$L_{\text{RG}} = \frac{L_0}{(\tau_q/\tau_0)^{2/3}} \quad \text{where} \quad L_0 = 1.2 \times 10^{12} \text{ m}^{-2}, \tau_0 = 100 \text{ ms} \quad (7)$$

we are thus led to expect that  $L_1 \approx 4 \times 10^{12} \text{ m}^2$ . Direct comparison of the calculated curves and measured data in Fig. 11 shows that this is plainly not the case. In fact, the data suggest that  $L_1$ , the vorticity created by the transition, is no more than  $10^{10} \text{ m}^{-2}$ , smaller than the expected value by at least two orders-of-magnitude.

## 8. DISCUSSION

Given the apparently positive outcome of the earlier investigations,<sup>8</sup> the null result of the present experiment has come as something of a surprise. There are several points to be made. First, Zurek did not expect his estimates of  $L_1$  be accurate to better than one, or possibly two, orders-of-magnitude, and his more recent estimate<sup>32</sup> suggests somewhat lower defect-densities. So it remains possible that his picture<sup>3,4,5,32</sup> is correct for  $^4\text{He}$  in all essential details, and that an improved experiment with faster expansions now being planned will reveal evidence of the Kibble-Zurek mechanism at work in liquid  $^4\text{He}$ . Secondly, it must be borne in mind that (2), and the value of  $\chi/B$  measured (from plots like those in Fig. 10) from the data of Fig. 9, refer to hydrodynamically generated vortex lines. Vorticity generated in the nonequilibrium phase transition might perhaps be significantly different, e.g. in respect of its loop-size distribution.<sup>33</sup> It could therefore decay



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faster, and might consequently be unobservable in the present experiments. Thirdly, it is surprising to us that the  $^3\text{He}$  experiments<sup>9,10</sup> seem to agree with Zurek's original estimates<sup>3,4,5</sup> whereas the present experiment shows that they overstate  $L_1$  by *at least* two orders-of-magnitude. It is not yet known for sure why this should be, although an interesting explanation of the apparent discrepancy has recently been suggested<sup>34</sup> by Karra and Rivers. They suggest that fluctuations near to  $T_\lambda$  may change the winding number, i.e. reduce the density of vortices produced in the  $^4\text{He}$  experiment, and they show that the analogous density reduction would be much smaller in the case of the  $^3\text{He}$  experiments.

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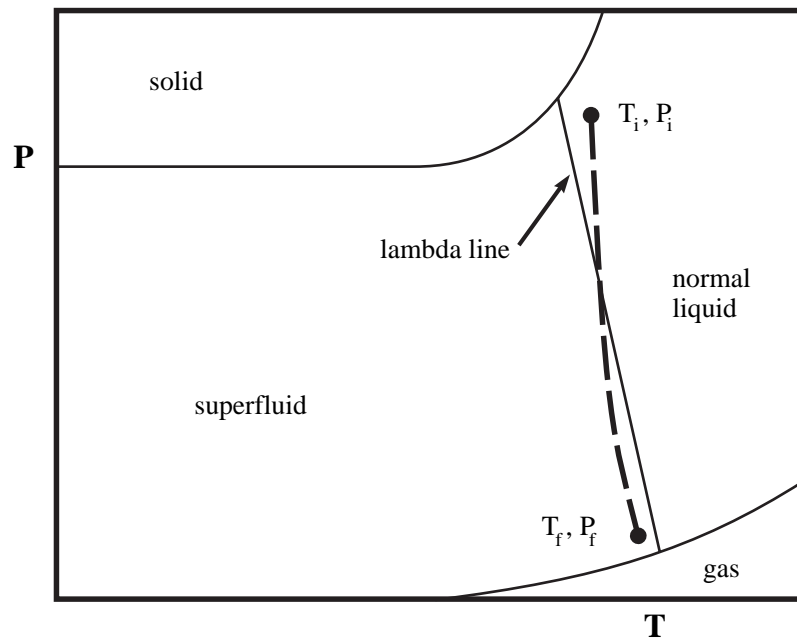


Fig. 1. Schematic of expansion trajectory through the  $^4\text{He}$  superfluid transition from a starting temperature and pressure  $(T_i, P_i)$  to final values  $(T_f, P_f)$ .

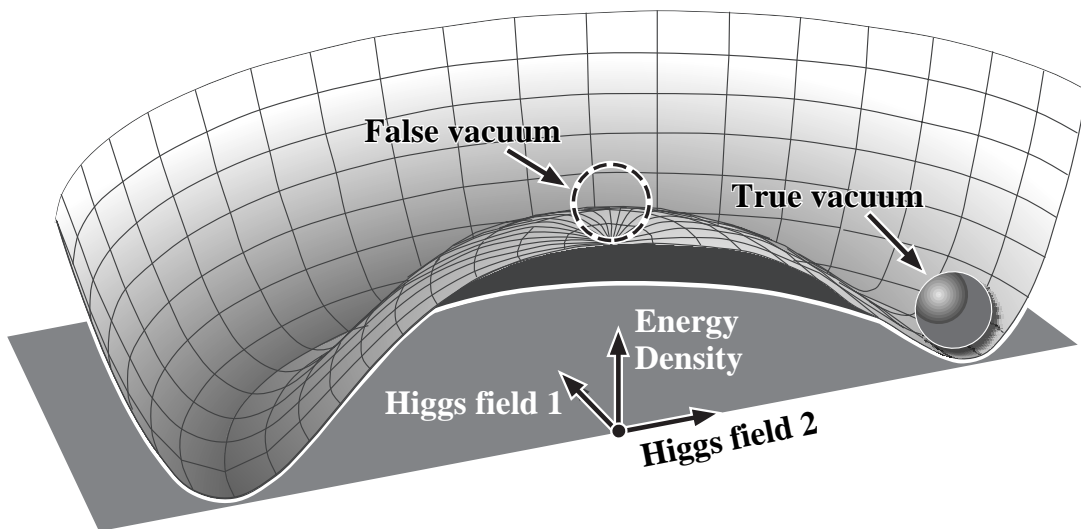


Fig. 2. Potential contribution to the free energy for the cosmological phase transition, after Guth and Steinhardt<sup>22</sup>.

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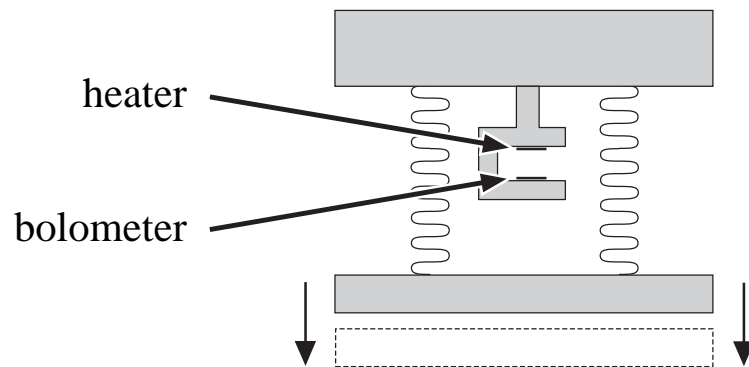


Fig. 3. Schematic diagram showing the main features of the original experimental cell. The heater and bolometer were mounted on a yoke immersed in the liquid  $^4\text{He}$ .

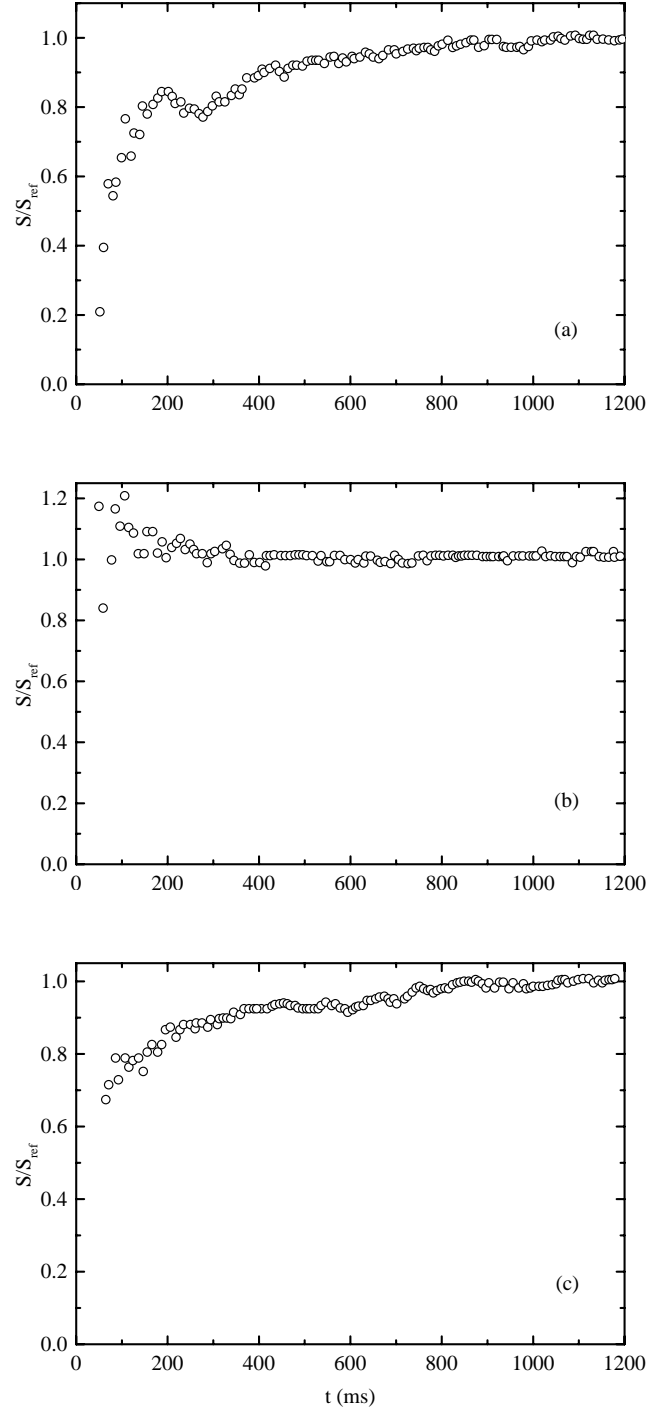


Fig. 4. Recovery of the scaled second-sound signal amplitude  $S$  as a function of time  $t$  in our original experiment, for various expansion trajectories: (a) through the lambda line,  $T_i = 1.81$  K,  $P_i = 29.6$  bar,  $T_f = 2.05$  K,  $P_f = 6.9$  bar; (b) starting well below the lambda line,  $T_i = 1.58$  K,  $P_i = 23.0$  bar,  $T_f = 1.74$  K,  $P_f = 4.0$  bar; (c) starting slightly ( $\sim 10$  mK) below the lambda line,  $T_i = 1.82$  K,  $P_i = 25.7$  bar,  $T_f = 2.03$  K,  $P_f = 6.9$  bar.

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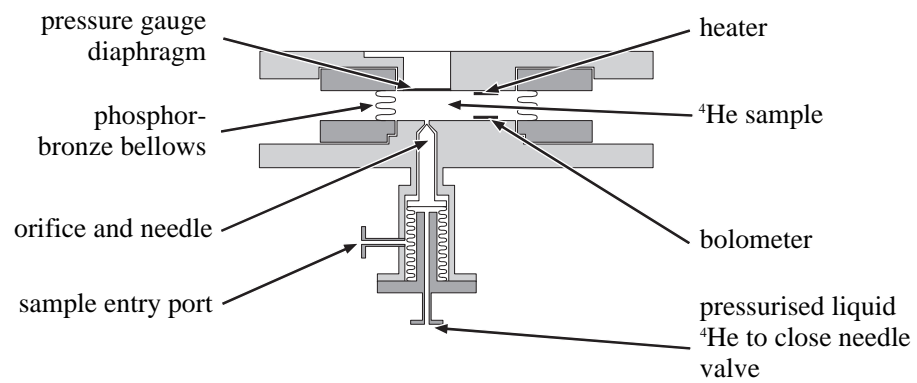


Fig. 5. The new expansion cell, designed so as to minimise hydrodynamic creation of vortices.

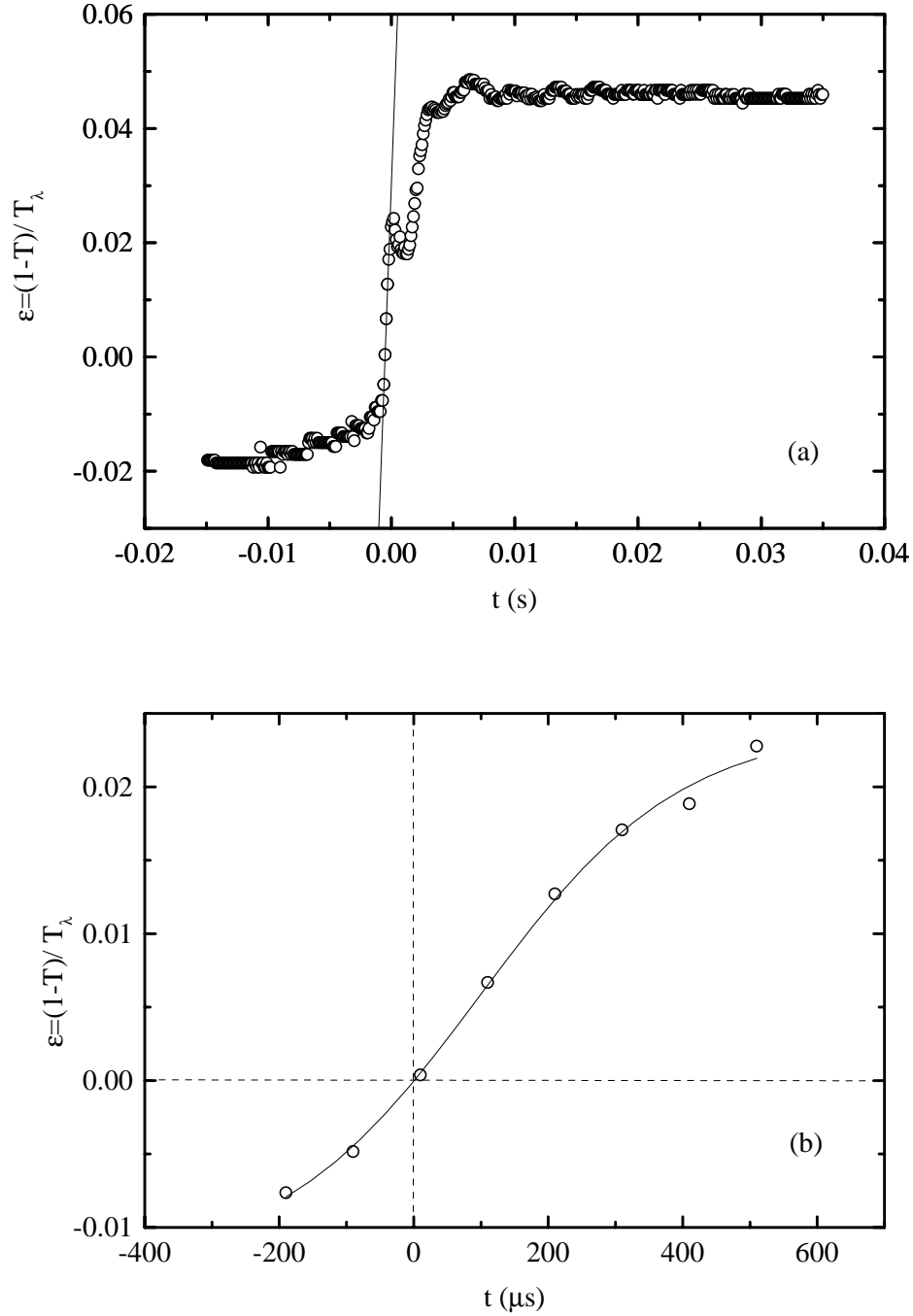


Fig. 6. Distance *versus* time  $t$  for a typical quench: (a) during the complete expansion; (b) enlargement of region near  $\epsilon = 0$ . The reciprocal of the gradient at the transition gives the value of the ‘quench time’ parameter,  $\tau_q$ .



## Expansion of Liquid $^4\text{He}$ Through the Lambda Transition

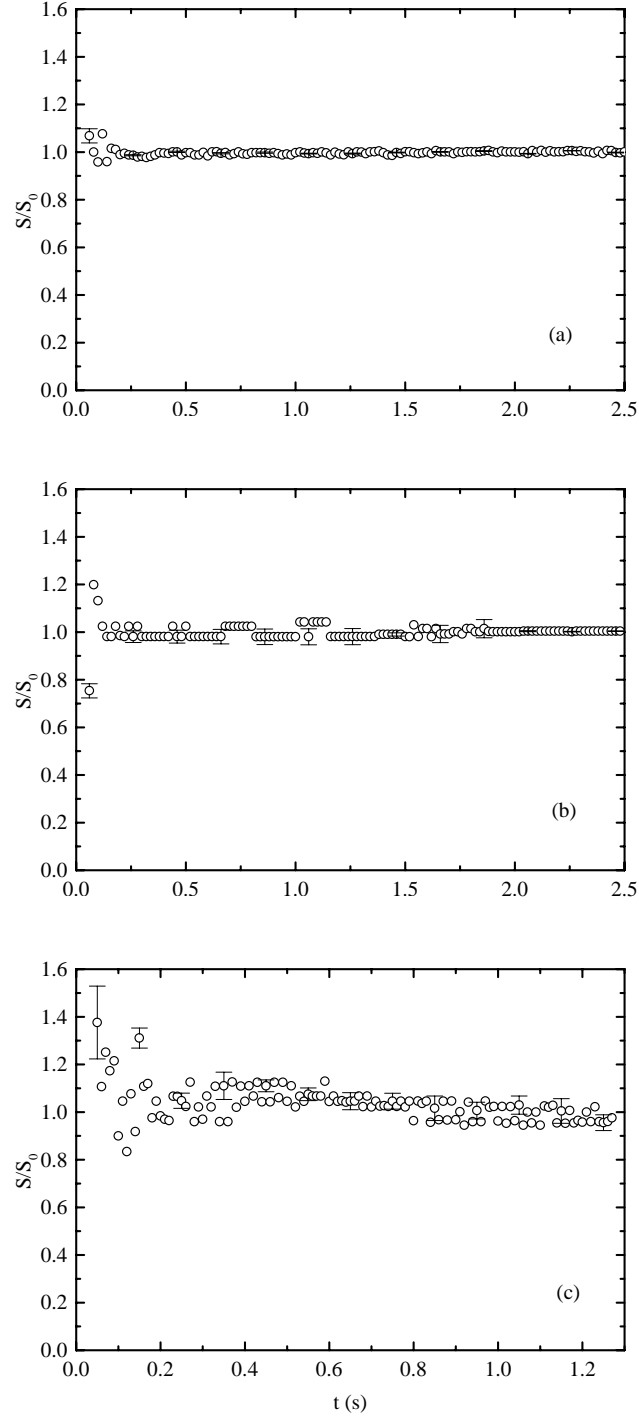


Fig. 7. Second-sound pulse amplitude  $S/S_0$  as a function of time following pressure quenches entirely within the helium-II phase: (a) starting far ( $\sim 490$  mK) below  $T_\lambda$  with  $T_i = 1.37$  K,  $P_i = 24.2$  bar,  $T_f = 1.47$  K,  $P_f = 7.2$  bar; (b) starting  $\sim 150$  mK below  $T_\lambda$  with  $T_i = 1.74$  K,  $P_i = 23.8$  bar,  $T_f = 1.76$  K,  $P_f = 6.9$  bar; (c) starting slightly ( $\sim 40$  mK) below  $T_\lambda$  with  $T_i = 1.85$  K,  $P_i = 22.4$  bar,  $T_f = 1.98$  K,  $P_f = 6.4$  bar.

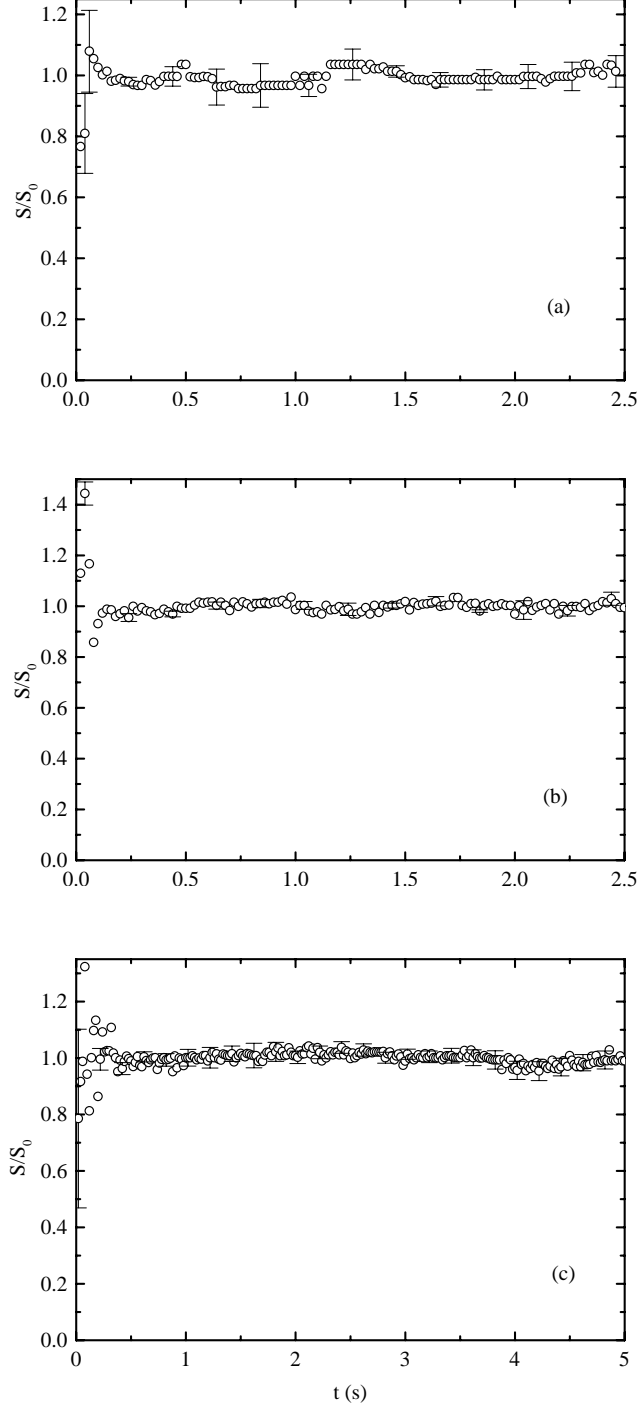


Fig. 8. Second-sound pulse amplitude  $S/S_0$  as a function of time following pressure quenches through the  $\lambda$  transition: (a) starting close to the transition, with  $T_i = 1.81$  K,  $P_i = 30.3$  bar,  $T_f = 2.03$  K,  $P_f = 6.2$  bar ( $\epsilon_i = -0.032$ ); (b) starting further above the transition, with  $T_i = 2.05$  K,  $P_i = 22.7$  bar,  $T_f = 2.09$  K,  $P_f = 5.9$  bar ( $\epsilon_i = -0.089$ ); (c) starting far above the transition, with  $T_i = 1.96$  K,  $P_i = 34.1$  bar,  $T_f = 2.07$  K,  $P_f = 6.1$  bar ( $\epsilon_i = -0.167$ ).

### Expansion of Liquid $^4\text{He}$ Through the Lambda Transition

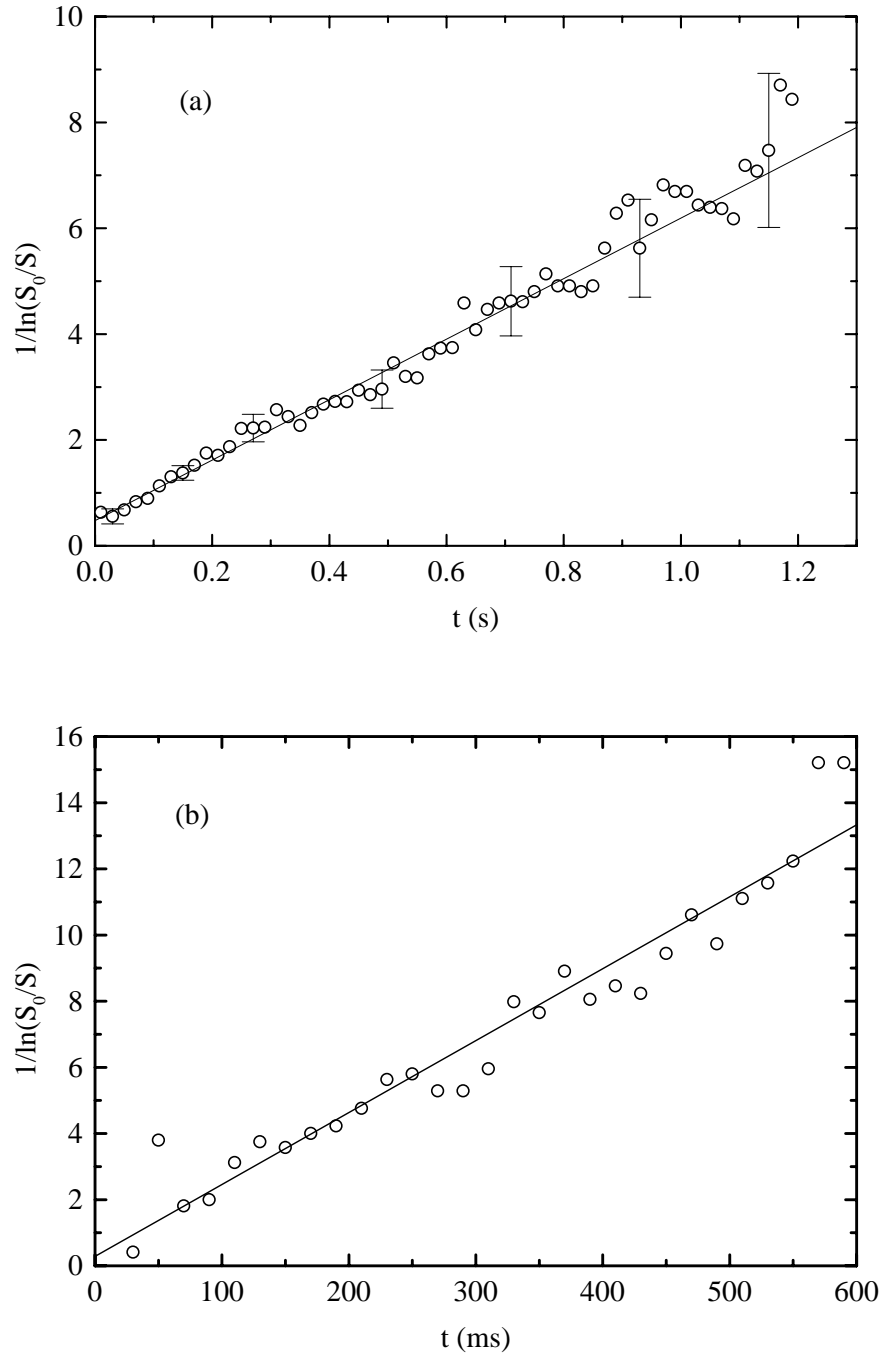


Fig. 9. Examples of  $(\ln S_0 - \ln S)^{-1}$  plotted *versus* time  $t$  for hydrodynamically created vortices.

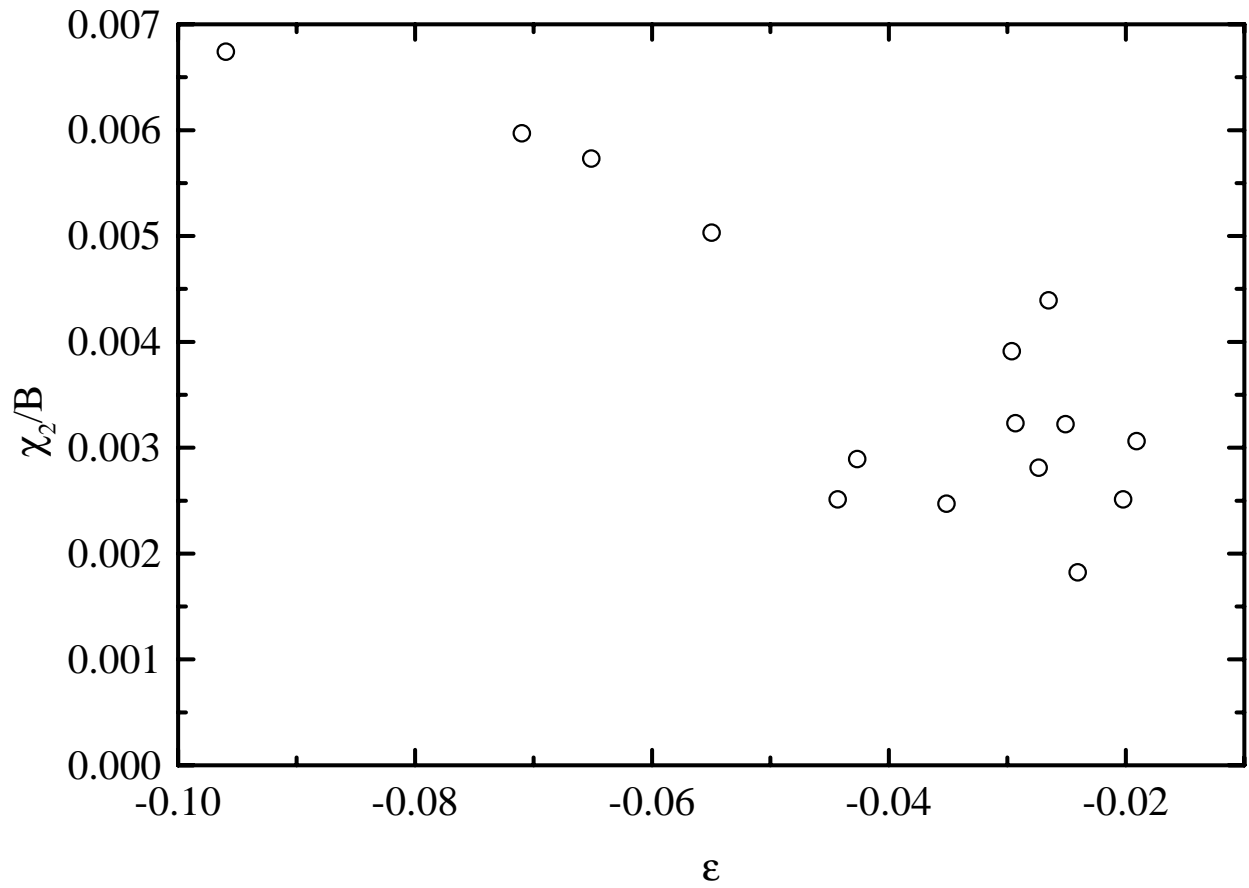


Fig. 10. Measured values of the parameter  $\chi_2/B$  as a function of distance  $\epsilon$  from the  $\lambda$ -transition.

## Expansion of Liquid $^4\text{He}$ Through the Lambda Transition

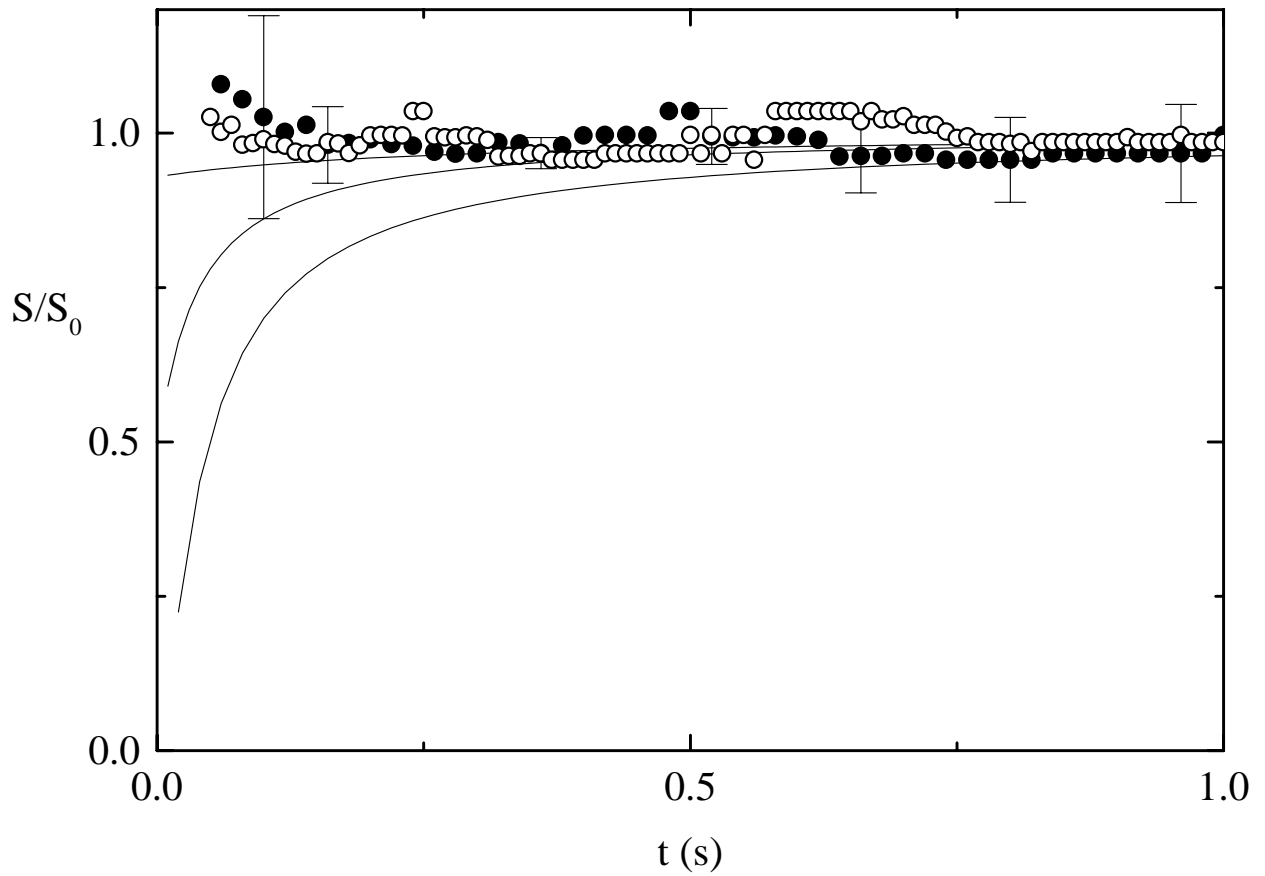


Fig. 11. Evolution of the second-sound amplitude  $S$  with time, following an expansion of the cell at  $t = 0$  (data points), normalised by its vortex-free value  $S_0$ . The open and closed symbols correspond to signal repetition rates of 100 and 50 Hz respectively, and in each case the starting and finishing conditions were  $\epsilon_i = -0.032$ ,  $\epsilon_f = 0.039$ . The curves refer to calculated signal evolutions for different initial line densities, from the bottom, of  $10^{12}$ ,  $10^{11}$  and  $10^{10} \text{ m}^{-2}$ .